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by Awawin Mustana Rohmah

Submission date: 25-Dec-2022 11:18PM (UTC-0700)
Submission ID: 1986643417
File name: full-paper-dinita_awin_revisi_2.pdf (464.94K)
Word count: 4405
Character count: 21409

Optimal Control and Sensitivity Analysis of Infectious Disease Spread in Two Regions Using Quarantine and Treatment

Dinita Rahmalia[a)] and Awawin Mustana Rohmah[b)]

Department of Mathematics, University of Islamic Darul Ulum Lamongan

[a)]*Corresponding author: dinitarahmalia@gmail.com*

[b)]*Electronic mail: awawin.emer@gmail.com*

Abstract. Infectious disease can spread among many regions so that it is required control for the prevention. In the dynamical model of spread of infectious disease in two regions, there are susceptible, exposed, infected, and recovered population and they move and travel each other. The novelties of this research are there are the addition control quarantine in first region and second region for reducing the number of individual moving and traveling and control treatment in first region and second region so that they can remove infected population to the recovered population. Moreover, the sensitivity analysis is also done for comparing the numerical simulation of each individual and optimal control applied. Optimal control is used for minimizing the number of exposed, infected individual in two regions and the cost of quarantine and treatment. In this research, comparison of numerical simulation without control and with quarantine and treatment control will be done. The method of computation of this problem is using Pontryagin's Maximum Principle with Forward Backward Sweep Method as numerical computation. Simulations have been done by various rate of movement for sensitivity analysis. Based on simulation results, quarantine and treatment can affect dynamical model of spread of infectious disease in two regions with the various rate of movement. From the comparison, the model with control quarantine and treatment can decrease exposed and infected individuals in both regions than the model without control. Control quarantine and treatment are also used for controling the infectious disease spread so that exposed and infected individuals become smaller.

INTRODUCTION

Nowadays, many infectious diseases have been found because the virus always mutate and form the new disease [13]. Healthy human can be infected by disease because doing contact through air, touching, droplet, and so on. The spreading of disease in the population, generally there are susceptible population, exposed population, infected population, and recovered population. Susceptible can be infected after contact with exposed or infected depends on transmission rate. Before being infected, the symptoms have not appeared yet so that susceptible becomes exposed. When the the virus mutate and symptoms have appeared, exposed becomes infected and then, infected becomes recovered [1,3].

Infectious disease can spread among many regions so that it is required control for the prevention. In this research, dynamical model of spread of infectious disease in two regions will be done mathematically. From previous research, mathematical models of some disease spread have been researched such as dengue disease [11], zika disease [12], avian flu disease [9], ebola disease [4,7] and cancer spread model [10].

In the dynamical model of spread of infectious disease in two regions, there are susceptible, exposed, infected, and recovered population and they move and travel each other. The novelties of this research are there are the addition control quarantine in first region and second region for reducing the number of individual moving and traveling and control treatment in first region and second region so that they can remove infected population to the recovered population. Moreover, the sensitivity analysis is also done for comparing the numerical simulation of each individual and optimal control applied. Optimal control is used for minimizing the number of exposed, infected individual in two regions and the cost of quarantine and treatment [5]. In this research, comparison of numerical simulation without control and with quarantine and treatment control will be done.

The method of computation of this problem is using Pontryagin's Maximum Principle with Forward Backward Sweep Method as numerical computation. Forward Backward Sweep Method compute solution of state variables forward with initial condition and solution of co-state variables backward with final condition [2]. Simulations have been done using various rate of movement for sensitivity analysis. Based on simulation results, quarantine and treatment can affect dynamical model of spread of infectious disease in two regions with the various rate of movement. From the comparison, the model with control quarantine and treatment can decrease exposed and infected individuals in both regions than the model without control. Control quarantine and treatment are also used for controling the infectious disease spread so that exposed and infected individuals become smaller.

DYNAMICAL MODEL

The dynamical model of spread of infectious disease in two regions will be explained in two parts. The first is the spread model of infectious disease without quarantine and treatment control and the second is the spread model of infectious disease with quarantine and treatment control.

Dynamical Model Without Control

The dynamical model of spread of infectious disease in two regions can be constructed in Fig. 1 in which there are first region and second region with each region there are susceptible population, exposed population, infected population, and recovered population that moving and traveling each other. Susceptible can be infected after contact with exposed or infected depends on transmission rate. Before being infected, the symptoms have not appeared yet so that susceptible becomes exposed. When the the virus mutate and symptoms have appeared, exposed becomes infected and then, infected becomes recovered [1,3]. The population included are : S_1 is the susceptible population in first region, S_2 is the susceptible population in second region, E_1 is the exposed population in first region, E_2 is the exposed population in second region, E_1 is the exposed population in first region, E_2 is the exposed population in second region, I_1 is the infected population in first region, I_2 is the infected population in second region, R_1 is the recovered population in first region, R_2 is the recovered population in second region.

The dynamical model can be modeled as system of differential equation as follows [6] :

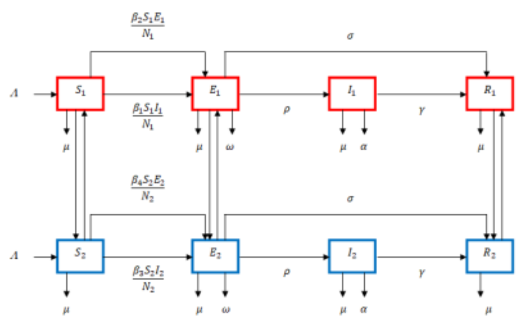

FIGURE 1. The dynamical model of spread of infectious disease in two regions

$$
\begin{aligned}
\frac{dS_1}{dt} &= \Lambda - \frac{\beta_1 S_1 I_1}{N_1} - \frac{\beta_2 S_1 E_1}{N_1} - \mu S_1 - p_{out} S_1 + p_{in} S_2 \\
\frac{dS_2}{dt} &= \Lambda - \frac{\beta_3 S_2 I_2}{N_2} - \frac{\beta_4 S_2 E_2}{N_2} - \mu S_2 - p_{in} S_2 + p_{out} S_1 \\
\frac{dE_1}{dt} &= \frac{\beta_1 S_1 I_1}{N_1} + \frac{\beta_2 S_1 E_1}{N_1} - \mu E_1 - \omega E_1 - \rho E_1 - \sigma E_1 - q_{out} E_1 + q_{in} E_2 \\
\frac{dE_2}{dt} &= \frac{\beta_3 S_2 I_2}{N_2} + \frac{\beta_4 S_2 E_2}{N_2} - \mu E_2 - \omega E_2 - \rho E_2 - \sigma E_2 - q_{in} E_2 + q_{out} E_1 \\
\frac{dI_1}{dt} &= \rho E_1 - \mu I_1 - \alpha I_1 - \gamma I_1 \\
\frac{dI_2}{dt} &= \rho E_2 - \mu I_2 - \alpha I_2 - \gamma I_2 \\
\frac{dR_1}{dt} &= \sigma E_1 + \gamma I_1 - \mu R_1 - r_{out} R_1 + r_{in} R_2 \\
\frac{dR_2}{dt} &= \sigma E_2 + \gamma I_2 - \mu R_2 - r_{in} R_2 + r_{out} R_1
\end{aligned}
\tag{1}
$$

with the positive solutions in all populations :

$$S_1(t) \geq 0, S_2(t) \geq 0, E_1(t) \geq 0, E_2(t) \geq 0, I_1(t) \geq 0, I_2(t) \geq 0, R_1(t) \geq 0, R_2(t) \geq 0 \tag{2}$$

The parameters used are :
Λ : the rate of birth
β_1 : the rate of disease transmission between susceptible and infected individual in first region
β_2 : the rate of disease transmission between susceptible and exposed individual in first region
β_3 : the rate of disease transmission between susceptible and infected individual in second region
β_4 : the rate of disease transmission between susceptible and exposed individual in second region
μ : the rate of natural death in common individual
ω : the rate of death by disease in exposed population
α : the rate of death by disease in infected population
ρ : the rate of mutation
σ : the rate of recovery in exposed population
γ : the rate of recovery in infected population
The susceptible, exposed, and recovered population of each region move and travel in each other with the rate p_{in}, p_{out} in susceptible population, q_{in}, q_{out} in exposed population, r_{in}, r_{out} in recovered population. The infected population can not move and travel because they should take rest or isolation for their healthy.

Dynamical Model with Quarantine and Treatment Control

Similar model to the dynamical model without control however there are the addition control quarantine $v_1(t)$ in susceptible of first region, quarantine $v_2(t)$ in susceptible of second region for reducing the number of individual moving and traveling. Actually there are quarantine $w_1(t)$ in exposed of first region and quarantine w_2 in exposed of second region. However, the quarantine of exposed in two regions should be applied massively so that $w_1(t) = w_2(t) = 1$ and causing $-q_{out}(1-w_1)E_1 + q_{in}(1-w_2)E_2 = 0, -q_{in}(1-w_2)E_2 + q_{out}(1-w_1)E_1 = 0$. Moreover, there are also control treatment $u_1(t)$ in infected of first region and treatment $u_2(t)$ in infected of second region so that they can remove infected population to the recovered population.
The system of differential equation in Eq.(1) can be modified in Eq.(3) respectively.

$$\begin{aligned}
\frac{dS_1}{dt} &= \Lambda - \frac{\beta_1 S_1 I_1}{N_1} - \frac{\beta_2 S_1 E_1}{N_1} - \mu S_1 - p_{out}(1 - v_1(t))S_1 + p_{in}(1 - v_2(t))S_2 \\
\frac{dS_2}{dt} &= \Lambda - \frac{\beta_3 S_2 I_2}{N_2} - \frac{\beta_4 S_2 E_2}{N_2} - \mu S_2 - p_{in}(1 - v_2(t))S_2 + p_{out}(1 - v_1(t))S_1 \\
\frac{dE_1}{dt} &= \frac{\beta_1 S_1 I_1}{N_1} + \frac{\beta_2 S_1 E_1}{N_1} - \mu E_1 - \omega E_1 - \rho E_1 - \sigma E_1 - q_{out} E_1 + q_{in} E_2 \\
\frac{dE_2}{dt} &= \frac{\beta_3 S_2 I_2}{N_2} + \frac{\beta_4 S_2 E_2}{N_2} - \mu E_2 - \omega E_2 - \rho E_2 - \sigma E_2 - q_{in} E_2 + q_{out} E_1 \\
\frac{dI_1}{dt} &= \rho E_1 - \mu I_1 - \alpha I_1 - \gamma I_1 - u_1(t) I_1 \\
\frac{dI_2}{dt} &= \rho E_2 - \mu I_2 - \alpha I_2 - \gamma I_2 - u_2(t) I_2 \\
\frac{dR_1}{dt} &= \sigma E_1 + \gamma I_1 - \mu R_1 - r_{out}(1 - v_1(t))R_1 + r_{in}(1 - v_2(t))R_2 + u_1(t) I_1 \\
\frac{dR_2}{dt} &= \sigma E_2 + \gamma I_2 - \mu R_2 - r_{in}(1 - v_2(t))R_2 + r_{out}(1 - v_1(t))R_1 + u_2(t) I_2
\end{aligned} \tag{3}$$

with the positive solutions in all populations :

$$S_1(t) \geq 0, S_2(t) \geq 0, E_1(t) \geq 0, E_2(t) \geq 0, I_1(t) \geq 0, I_2(t) \geq 0, R_1(t) \geq 0, R_2(t) \geq 0 \tag{4}$$

METHODS

The method of computation of this problem is using Pontryagin's Maximum Principle with Forward Backward Sweep Method as numerical computation.

Pontryagin's Maximum Principle

The purpose of optimal control problem in this problem is finding optimal $u_1(t), u_2(t), v_1(t), v_2(t)$ minimizing performance index in Eq.(5). Beside that, the number of exposed and infected individual in two regions are also minimized.

$$\min J = \int_{t=0}^{tf} A_1 E(t)_1^2 + A_2 E(t)_2^2 + B_1 I(t)_1^2 + B_2 I(t)_2^2 + C_1 u(t)_1^2 + C_2 u(t)_2^2 + D_1 v(t)_1^2 + D_2 v(t)_2^2 \tag{5}$$

The range of control $u_1(t), u_2(t), v_1(t), v_2(t)$ is between 0-1 as effectiveness rate. Score 0 represents the control is fail or not to be applied while score 1 represents the control is success or applied entirely. In quarantine control is the cost spent by government for reducing the movement to the other region. In treatment control is the drug cost for removing infected population to the recovered population.

For effectiveness of quarantine $u_1(t), u_2(t)$ applied in susceptible and recovered individuals, we will give example as follows. Suppose that there are 10 individuals. If the quarantine control is 0.2 (20%), then individuals who are out of quarantine and travel to other region are $(1-0.2)\times 10 = 8$ individuals and the individuals who are keep stay in own region are 2 individuals. However if the quarantine control is 0.8 (80%), then individuals who are out of quarantine and travel to other region are $(1-0.8)\times 10 = 2$ individuals and the individuals who are keep stay in own region are 8 individuals. It describes that smaller effectiveness of quarantine causes individuals who are out of quarantine are larger and individuals who are keep stay in own region are smaller and vice versa.

For effectiveness of treatment $v_1(t), v_2(t)$ applied in infected individuals, we will give example as follows. Suppose that there are 10 infected individuals. If the treatment control is 0.2 (20%), then infected individuals who are recovered and remove to recovered individuals are $0.2\times 10 = 2$ individuals. If the treatment control is 0.8 (80%), then infected individuals who are recovered and remove to recovered individuals are $0.8\times 10 = 8$ individuals. It describes that smaller effectiveness of treatment causes individuals who are recovered are smaller and vice versa.

$A_1 \geq 0, A_2 \geq 0, B_1 \geq 0, B_2 \geq 0, C_1 \geq 0, C_2 \geq 0, D_1 \geq 0, D_2 \geq 0$, is the weights of performance index.

From performance index and dynamical model with control as state model, Hamiltonian equation are in Eq.(6).

$$\begin{aligned}
H \;=\; & A_1 E(t)_1^2 + A_2 E(t)_2^2 + B_1 I(t)_1^2 + B_2 I(t)_2^2 + C_1 u(t)_1^2 + C_2 u(t)_2^2 + D_1 v(t)_1^2 + D_2 v(t)_2^2 \\
& + \lambda_1 (\Lambda - \frac{\beta_1 S_1 I_1}{N_1} - \frac{\beta_2 S_1 E_1}{N_1} - \mu S_1 - p_{out}(1 - v_1(t)) S_1 + p_{in}(1 - v_2(t)) S_2) \\
& + \lambda_2 (\Lambda - \frac{\beta_3 S_2 I_2}{N_2} - \frac{\beta_4 S_2 E_2}{N_2} - \mu S_2 - p_{in}(1 - v_2(t)) S_2 + p_{out}(1 - v_1(t)) S_1) \\
& + \lambda_3 (\frac{\beta_1 S_1 I_1}{N_1} + \frac{\beta_2 S_1 E_1}{N_1} - \mu E_1 - \omega E_1 - \rho E_1 - \sigma E_1 - q_{out} E_1 + q_{in} E_2) \\
& + \lambda_4 (\frac{\beta_3 S_2 I_2}{N_2} + \frac{\beta_4 S_2 E_2}{N_2} - \mu E_2 - \omega E_2 - \rho E_2 - \sigma E_2 - q_{in} E_2 + q_{out} E_1) \\
& + \lambda_5 (\rho E_1 - \mu I_1 - \alpha I_1 - \gamma I_1 - u_1(t) I_1) \\
& + \lambda_6 (\rho E_2 - \mu I_2 - \alpha I_2 - \gamma I_2 - u_2(t) I_2) \\
& + \lambda_7 (\sigma E_1 + \gamma I_1 - \mu R_1 - r_{out}(1 - v_1(t)) R_1 + r_{in}(1 - v_2(t)) R_2 + u_1(t) I_1) \\
& + \lambda_8 (\sigma E_2 + \gamma I_2 - \mu R_2 - r_{in}(1 - v_2(t)) R_2 + r_{out}(1 - v_1(t)) R_1 + u_2(t) I_2)
\end{aligned} \tag{6}$$

If $u_1^*(t), u_2^*(t), v_1^*(t), v_2^*(t)$ are optimal control corresponding state system, there exist co-state variables : $\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t), \lambda_5(t), \lambda_6(t), \lambda_7(t), \lambda_8(t)$ satisfying the following :

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= -\frac{\partial H}{\partial S_1}, \frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial S_2} \\
\frac{d\lambda_3}{dt} &= -\frac{\partial H}{\partial E_1}, \frac{d\lambda_4}{dt} = -\frac{\partial H}{\partial E_2} \\
\frac{d\lambda_5}{dt} &= -\frac{\partial H}{\partial I_1}, \frac{d\lambda_6}{dt} = -\frac{\partial H}{\partial I_2} \\
\frac{d\lambda_7}{dt} &= -\frac{\partial H}{\partial R_1}, \frac{d\lambda_8}{dt} = -\frac{\partial H}{\partial R_2}
\end{aligned} \tag{7}$$

with the final conditions $\lambda_{1,2,3,4,5,6,7,8}(tf) = 0$.

And the optimal controls are :

$$
\begin{aligned}
\frac{\partial H}{\partial u_1} &= 2C_1u_1 + \lambda_5(-I_1) + \lambda_7(I_1) = 0 \\
u_1(t) &= \min(1, \max(0, \frac{\lambda_5 I_1 - \lambda_7 I_1}{2C_1}))
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\frac{\partial H}{\partial u_2} &= 2C_2u_2 + \lambda_6(-I_2) + \lambda_8(I_2) = 0 \\
u_2(t) &= \min(1, \max(0, \frac{\lambda_6 I_2 - \lambda_8 I_2}{2C_2}))
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\frac{\partial H}{\partial v_1} &= 2D_1v_1 + \lambda_1(p_{out}S_1) + \lambda_2(-p_{out}S_1) + \lambda_7(r_{out}R_1) + \lambda_8(-r_{out}R_1) = 0 \\
v_1(t) &= \min(1, \max(0, \frac{-\lambda_1(p_{out}S_1) + \lambda_2(p_{out}S_1) - \lambda_7(r_{out}R_1) + \lambda_8(r_{out}R_1)}{2D_1}))
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\frac{\partial H}{\partial v_2} &= 2D_2v_2 + \lambda_1(-p_{in}S_2) + \lambda_2(p_{in}S_2) + \lambda_7(-r_{in}R_2) + \lambda_8(r_{in}R_2) = 0 \\
v_2(t) &= \min(1, \max(0, \frac{\lambda_1(p_{in}S_2) - \lambda_2(p_{in}S_2) + \lambda_7(r_{in}R_2) - \lambda_8(r_{in}R_2)}{2D_2}))
\end{aligned} \tag{11}
$$

Forward Backward Sweep Method

Forward backward sweep method that is applied on optimal control of dynamical model of spread of infectious disease in two regions can be designed as follows [2]:
Suppose state variables and co-state variables are :

$$
\begin{aligned}
f_1 &= \frac{dS_1}{dt}, f_2 = \frac{dS_2}{dt}, f_3 = \frac{dE_1}{dt}, f_4 = \frac{dE_2}{dt}, f_5 = \frac{dI_1}{dt}, f_6 = \frac{dI_2}{dt}, f_7 = \frac{dR_1}{dt}, f_8 = \frac{dR_2}{dt} \\
g_1 &= \frac{d\lambda_1}{dt}, g_2 = \frac{d\lambda_2}{dt}, g_3 = \frac{d\lambda_3}{dt}, g_4 = \frac{d\lambda_4}{dt}, g_5 = \frac{d\lambda_5}{dt}, g_6 = \frac{d\lambda_6}{dt}, g_7 = \frac{d\lambda_7}{dt}, g_8 = \frac{d\lambda_8}{dt}
\end{aligned} \tag{12}
$$

1. Computing solution of state variables forward with initial condition :
   $x(0) = (S_1(0), S_2(0), E_1(0), E_2(0), I_1(0), I_2(0), R_1(0), R_2(0))$ using Runge Kutta

$$
\begin{aligned}
k_{1i} &= f_i(t, x(t), u(t), v(t)), i = 1, 2, \ldots, 8 \\
k_{2i} &= f_i(t + \frac{h}{2}, x(t) + \frac{h}{2}k_1, \frac{u(t) + u(t+h)}{2}, \frac{v(t) + v(t+h)}{2}), i = 1, 2, \ldots, 8 \\
k_{3i} &= f_i(t + \frac{h}{2}, x(t) + \frac{h}{2}k_2, \frac{u(t) + u(t+h)}{2}, \frac{v(t) + v(t+h)}{2}), i = 1, 2, \ldots, 8 \\
k_{4i} &= f_i(t + h, x(t) + hk_3, u(t+h), v(t+h)), i = 1, 2, \ldots, 8 \\
x_i(t+h) &= x_i(t) + \frac{h}{6}(k_{1i} + 2k_{2i} + 2k_{3i} + k_{4i}), i = 1, 2, \ldots, 8
\end{aligned} \tag{13}
$$

2. Computing solution of c0-state variables backward with final condition
   $\lambda(tf) = (\lambda_1(tf), \lambda_2(tf), \lambda_3(tf), \lambda_4(tf), \lambda_5(tf), \lambda_6(tf), \lambda_7(tf), \lambda_8(tf))$ using Runge Kutta

$$
\begin{aligned}
k_{1i} &= g_i(t, x(t), \lambda_i(t), u(t), v(t)), i = 1, 2, \ldots, 8 \\
k_{2i} &= g_i(t + \frac{h}{2}, \frac{x(t) + x(t-h)}{2}, \lambda(t) - \frac{h}{2}k_1, \frac{u(t) + u(t-h)}{2}, \frac{v(t) + v(t-h)}{2}), i = 1, 2, \ldots, 8 \\
k_{3i} &= g_i(t - \frac{h}{2}, \frac{x(t) + x(t-h)}{2}, \lambda(t) - \frac{h}{2}k_2, \frac{u(t) + u(t-h)}{2}, \frac{v(t) + v(t-h)}{2}), i = 1, 2, \ldots, 8 \\
k_{4i} &= g_i(t - h, x(t-h), \lambda(t) - hk_3, u(t-h), v(t-h)), i = 1, 2, \ldots, 8 \\
\lambda_i(t-h) &= \lambda_i(t) - \frac{h}{6}(k_{1i} + 2k_{2i} + 2k_{3i} + k_{4i}), i = 1, 2, \ldots, 8
\end{aligned} \tag{14}
$$

3. Computing optimal control $u_1^*(t), u_2^*(t), v_1^*(t), v_2^*(t)$ using Eq.(8)-(11) respectively
4. Updating optimal control

$$u_1 \leftarrow \frac{u_1 + u_{1,old}}{2}, u_2 \leftarrow \frac{u_2 + u_{2,old}}{2}, v_1 \leftarrow \frac{v_1 + v_{1,old}}{2}, v_2 \leftarrow \frac{v_2 + v_{2,old}}{2} \tag{15}$$

5. Repeat the procedure until converge

RESULTS

The simulations are compared between model without control and model with quarantine and treatment control. Parameter used in first region and second region are similar except the rate of disease transmission. Parameter used are : the rate of birth Λ is 3, the rate of disease transmission between susceptible and infected individual in first region β_1 is 0.3, the rate of disease transmission between suscpetible and exposed individual in first region β_2 is 0.3, the rate of disease transmission between susceptible and infected individual in second region β_3 is 0.03, the rate of disease transmission between susceptible and exposed individual in second region β_4 is 0.03, the rate of natural death in common individual μ is 0.0001, the rate of death by disease in exposed population ω is 0.002, the rate of death by disease in infected population α is 0.003, the rate of mutation ρ is 1/14, the rate of recovery in exposed population σ is 0.08, the rate of recovery in infected population γ is 0.05[6].
For each simulation, it will be done sensitivity analysis using various rate of p_{in}, p_{out} with value 0, 0.005, 0.010, 0.015, 0.020, 0.025, various rate of q_{in}, q_{out} with value 0, 0.002, 0.004, 0.006, 0.008, 0.010, various rate of r_{in}, r_{out} with value 0, 0.003, 0.006, 0.009, 0.012, 0.015.

The Spread Model of Infectious Disease Without Control

The simulation of the spread model of infectious disease in two regions without quarantine and treatment control can be seen in Fig. 2, Fig. 3, Fig. 4, and Fig. 5.
Fig. 2 is numerical simulation of susceptible population in first region (top) and second region (bottom) with the various rate of $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$. They seem that the number of susceptible individual decrease because infectious diseases making susceptible individuals become exposed individuals.
Fig. 3 is numerical simulation of exposed population in first region (top) and second region (bottom) with the various

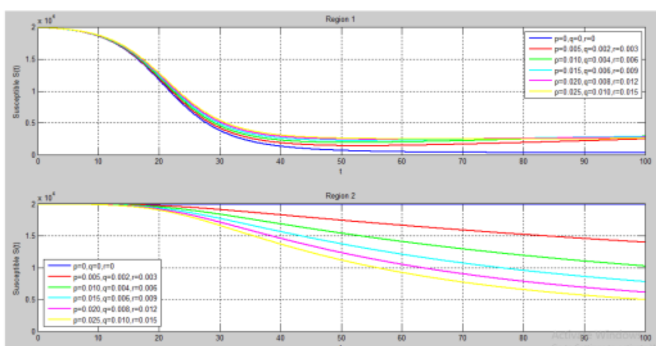

FIGURE 2. Numerical simulation of susceptible population in two regions without control

rate of $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$. Fig. 4 is numerical simulation of infected population in first region (top) and second region (bottom) with the various rate of $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$. They seem that the number of individual increase because they are attacked infectious disease then decrease because they are removed due to dead or recover. In both of first region and second region, higher $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$ make the number of exposed and infected individual higher.

Fig. 5 is numerical simulation of recovered population in first region (top) and second region (bottom) with the vari-

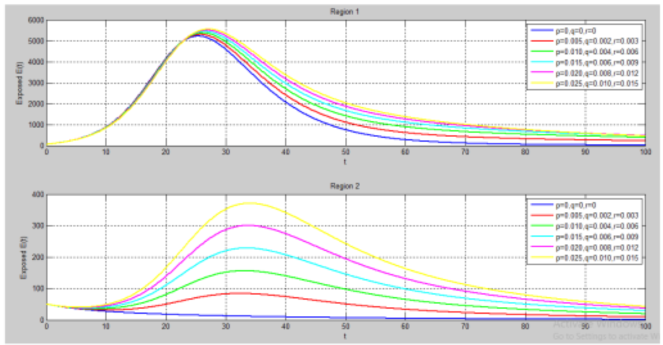

FIGURE 3. Numerical simulation of exposed population in two regions without control

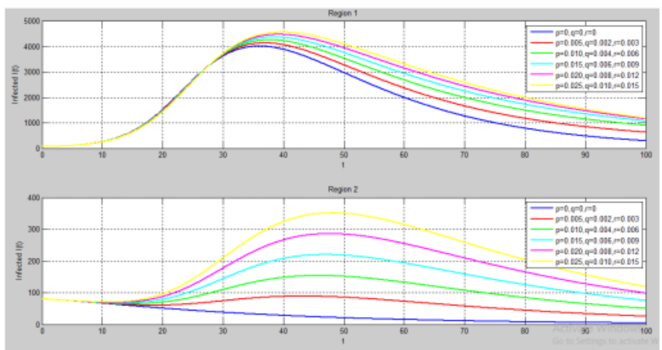

FIGURE 4. Numerical simulation of infected population in two regions without control

ous rate of $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$. They seem that in both of first region and second region the number of individual increase because there are many individuals who are recover even from exposed population or infected population.

The Spread Model of Infectious Disease with Quarantine and Treatment Control

The simulation of the spread model of infectious disease in two regions with quarantine and treatment control can be seen in Fig. 6, Fig. 7, Fig. 8, and Fig. 9. Optimal control of quarantine can be seen in Fig. 10 and optimal control of treatment can be seen in Fig. 11.

Fig. 6 is numerical simulation of susceptible population in first region (top) and second region (bottom) with the various rate of $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$. They seem that in first region, the number of susceptible individual decrease because quarantine effect and in second region, the number of susceptible individual increase because quarantine effect.

Fig. 7 is numerical simulation of exposed population in first region (top) and second region (bottom) with the various rate of $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$. They seem that in first region, the number of exposed individual increase and then decrease because fully quarantine effect and in second region, the number of exposed individual decrease and approach to zero because fully quarantine effect.

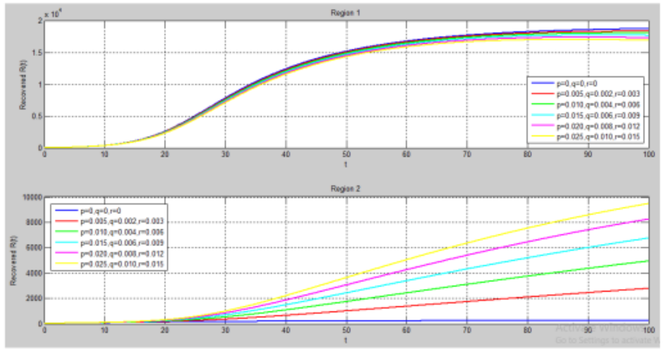

FIGURE 5. Numerical simulation of recovered population in two regions without control

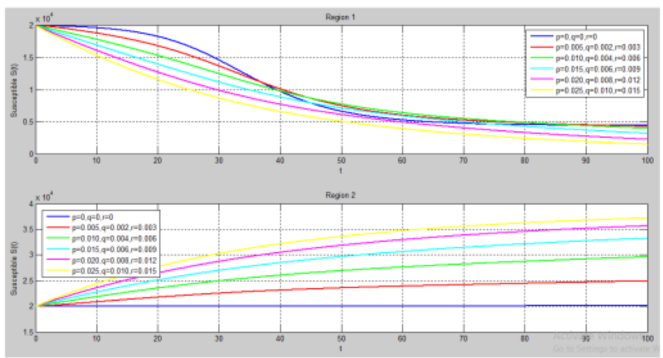

FIGURE 6. Numerical simulation of susceptible population in two regions with control

Fig. 8 is numerical simulation of infected population in first region (top) and second region (bottom) with the various rate of $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$. They seem that in both of first region and second region, the number of infected individual has gone and approaches to zero because the effects of treatment.

Fig. 9 is numerical simulation of recovered population in first region(top) and second region (bottom) with the various rate of $p_{in}, p_{out}, q_{in}, q_{out}, r_{in}, r_{out}$. They seem that in both of first region and second region the number of individual increase because there are many individuals who are recover because the treatment.

Fig. 10 is the numerical solution of optimal control of quarantine and Fig. 11 is the numerical solution of optimal control of treatment. Effectiveness rate is between 0-1. Score 0 represents the control is fail or not to be applied while score 1 represents the control is success or applied entirely. In quarantine control is the cost spent by government for reducing the movement to the other region. It describes that smaller effectiveness of quarantine causes individuals who are out of quarantine are larger and individuals who are keep stay in own region are smaller and vice versa. In treatment control is the drug cost for removing infected population to the recovered population. It describes that smaller effectiveness of treatment causes individuals who are recovered are smaller and vice versa. From the Fig. 11, in first time, the treatment (drug cost) is applied massively but in the next time, the treatment (drug cost) is not applied because there are no infected individuals.

From the comparison, the model with control quarantine and treatment can decrease exposed and infected individuals in both regions than the model without control. Control quarantine and treatment are also used for controlling the infectious disease spread so that exposed and infected individuals become smaller.

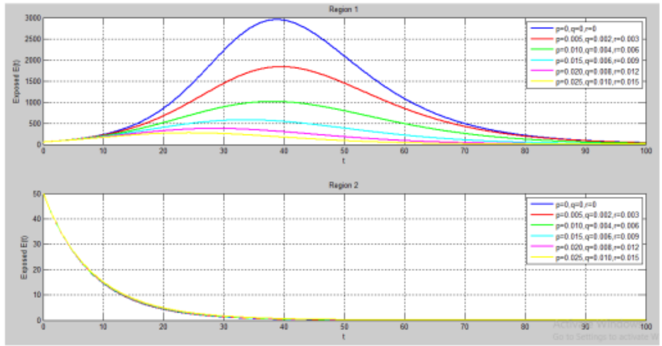

FIGURE 7. Numerical simulation of exposed population in two regions with control

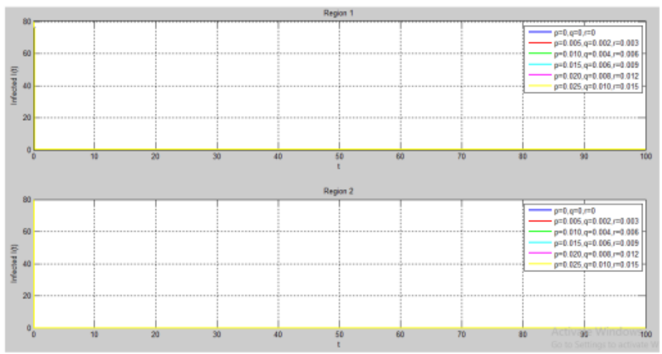

FIGURE 8. Numerical simulation of infected population in two regions with control

CONCLUSION

In the dynamical model of spread of infectious disease in two regions, there are susceptible, exposed, infected, and recovered population and they move and travel each other. There are the addition control quarantine in first region and second region for reducing the number of individual moving and traveling and control treatment in first region and second region so that they can remove infected population to the recovered population. Optimal control is used for minimizing the number of exposed, infected individual in two regions and the cost of quarantine and treatment. The method of computation of this problem is using Pontryagin's Maximum Principle with Forward Backward Sweep Method as numerical computation. Simulations have been done using various rate of movement for sensitivity analysis. Based on simulation results, quarantine and treatment can affect dynamical model of spread of infectious disease in two regions with the various rate of movement. From the comparison, the model with control quarantine and treatment can decrease exposed and infected individuals in both regions than the model without control. Control quarantine and treatment are also used for controlling the infectious disease spread so that exposed and infected individuals become smaller.

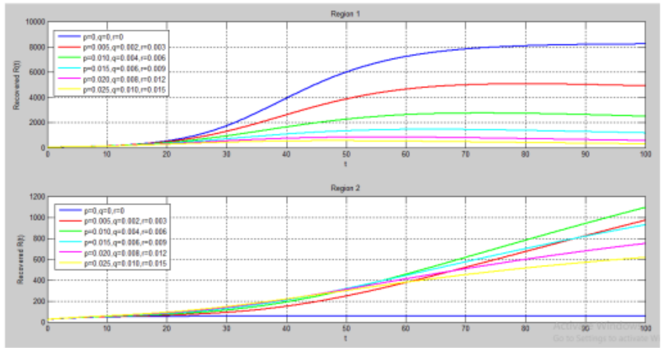

FIGURE 9. Numerical simulation of recovered population in two regions with control

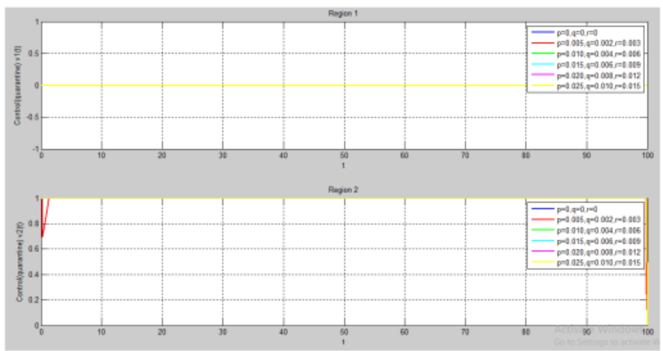

FIGURE 10. Optimal control of quarantine

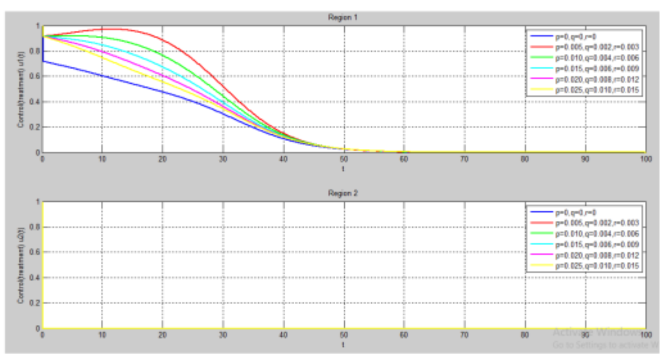

FIGURE 11. Optimal control of treatment

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ACKNOWLEDGMENTS

High appreciation to the Ministry of Education and Culture (Kemendikbud), Republic of Indonesia for the Penelitian Dosen Pemula fund for the research conducted in the year of 2020.

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